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Promising Practices in Elementary School Mathematics

by

RONALD C. WELCH

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BULLETIN OF THE SCHOOL OF EDUCATION
INDIANA UNIVERSITY

PROMISING PRACTICES IN ELEMENTARY SCHOOL MATHEMATICS

by

Ronald C. Welch

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Mathematics can be fun! (Hammond Public Schools)

FOREWORD

Significant concerns are being expressed about the teaching of mathematics in the elementary school. Recent scientific and technological developments have "spotlighted" our existing mathematics programs. Developments in the field of mathematics itself and in its importance to intelligent participation in our society call urgently for broader vision and deeper understanding of a modern mathematics curriculum.

This bulletin is addressed to all teachers who seek to help their pupils develop a breadth and depth of understanding in mathematics.

Significantly, the practices described in this bulletin are taken from actual classroom situations reported by selected public schools in Indiana. The introductory material preceding each topic is a summary of important research findings, principles, and points of view related to a modern elementary school mathematics program. Selected references are provided for those who wish to consult additional materials related to each topic.

During the preparation of this bulletin the author became indebted to many people. In acknowledging the assistance of over 350 teachers and over 50 schools which participated in this study, he hesitates to attempt to mention individual names because of the possibility of inadvertantly omitting one that should be included. He does wish, however, to say that special credit is due the following persons because of their responsibilities in selecting and contacting teachers to participate in this study: Lora L. Batchelor, Bloomington Metropolitan Schools; L. Frances Smith, Director of Elementary Education, Columbus Community Schools; Anne Hopman and Marie Kupres, Directors of Elementary Education, Hammond Public Schools; Vera W. Hopping, Supervisor of Elementary Education, Indianapolis Public Schools; Helen McDaniel, Elementary Director, Madison Consolidated Schools; William Paul Moser, Director of Elementary Education, New Albany-Floyd County Consolidated School Corporation; and Ruth E. Staninger, Director of Instruction, Kindergarten and Elementary Schools, Vincennes Public Schools.

Recognition is given to Richard DeGraff, Graduate Assistant, School of Education, Indiana University, for his assistance in doing some of the research for the introductory material and list of selected references.

Acknowledgment is given to Kathleen Dugdale, Editor of Publications in Education at Indiana University, for editing the final copy of the manuscript.

Ronald C. Welch

The "World of Mathematics" is at his finger tips as this sixth grade pupil works with geometric shapes and designs. (Hammond Public Schools)

INTRODUCTION

The mathematics program for the elementary school has been too narrowly conceived, and arithmetic, as it is usually thought of, has been too narrowly defined. Mere computation with numbers limits the vision to one narrow aspect of the science of mathematics.

For decades a majority of elementary school programs have emphasized speed, accuracy, and efficiency in computation. As a result, other facets of the science of mathematics have often been either omitted or given only token attention. Arithmetic programs have too often been characterized as simply drill or skill programs.

From past and present scientific and technological achievements it seems obvious that the development of a high level of quantitative thinking is needed. Mathematics, stripped of its system, relationships, and social significance, does not meet this need. Rote memorization and mechanical computation deals only with a "bone-pile" of technicalities. Instead of improving their powers to reason and think quantitatively, our children become susceptible to the disease of "computationitis."

Fortunately more and more of our elementary schools are developing modern mathematics programs. There is increasing evidence of

curricula more closely geared to the needs of contemporary society. Such programs place emphasis upon understanding, creativity, discovery, and experimentation. Children's needs, interests, and aptitudes are challenged by the provision of rich, vital, and well-integrated learning experiences. Quantitative thinking is thought of as a major outcome of such learning experiences. Computation is assuming its proper role as a tool, as a means rather than an end product of mathematics in the elementary school.

Purpose

Promising practices are emerging from some of the more broadly conceived elementary school mathematics programs. It is the purpose of this bulletin to identify, select, and present some of these practices. More specifically, the bulletin is designed to:

1. Present important but oftentimes neglected or misconceived aspects of a modern mathematics program

2. Develop important background information and understandings of each of the selected aspects of a modern mathematics program

3. Describe actual classroom practices and procedures illustrating each of the selected aspects of a modern mathematics program

4. Analyze the described classroom practices and procedures and present generalizations designed to help teachers understand better the illustrative ideas, techniques, and devices which make mathematical behavior functional.

Procedure

Two criteria were used for the selection of schools to be used in this study: (1) The schools within the systems should contain evidence of the development of a more broadly conceived mathematics program in which promising practices are taking place; and (2) the schools should be located in Indiana and should be so situated geographically that other teachers can visit them and identify themselves with these promising practices.

Seven school systems were selected: Bloomington, Columbus, Hammond, Indianapolis, Madison, New Albany, and Vincennes. The elementary supervisor of each of these seven school systems was contacted and asked to select 15 to 20 teachers in that system whose classrooms contained promising practices in mathematics. Each of these selected teachers was sent a letter requesting help in identifying and describing promising practices in one or more of the following aspects of a modern mathematics program:

1. Using concrete and semi-concrete instructional aids

2. Promoting problem-solving ability

3. Promoting quantitative vocabulary and reading skills

4. Making, using, and interpreting graphs

5. Teaching history and development of our number system

6. Using instructional aids centers and bulletin boards

7. Integrating mathematics with other subject areas

8. Using units of work.

In addition, each supervisor and each teacher was asked to send photographs which might help to illustrate the described practices.

Photographs and descriptions provided by the supervisors and teachers were carefully screened and analyzed, and representative materials on each of the topics were selected to be included in the bulletin.

The generalizations which follow each of the described practices are of two types. When the selections are in the form of specific lessons, the generalizations designate possible outcomes of those particular lessons. When the selections are general descriptions of classroom practices, broad generalizations are set forth stressing the importance and significance of such classroom procedures. Both types of generalizations are designed to help the teacher better interpret the ideas, techniques, and devices described in this bulletin.

USING CONCRETE AND SEMI-CONCRETE INSTRUCTIONAL AIDS

A modern elementary classroom should be regarded as a learning laboratory. Among the various kinds of materials it contains should be those that enable children to explore, experiment with, and discover number facts and number relationships.

Fourth grade children learn about liquid measurement through experimentation with real materials.
(Vincennes Public Schools)

Three kinds of instructional aids are essential for an effective mathematics program. These are often classified as concrete, semi-concrete, and symbolic. Concrete instructional aids are those that children can manipulate and move about, such as counters or tens blocks. With such materials children can discover principles and relationships not fully comprehended from just a formal verbal presentation.

Strip films, charts, diagramatic representations, and pictures are usually classified as semi-concrete materials. These materials enable the learner to move toward a more abstract visualization and understanding of number facts and number solutions.

Little needs to be said about symbolic instructional aids. These are the most commonly used types of materials. Textbook explanations, lists of examples, and verbal problems are most familiar to all children and teachers. Therefore, attention should be concentrated upon concrete and semi-concrete instructional aids.

The correct use of instructional aids determines the value of equipping a classroom with a variety of these aids. The mere possession of many types of learning materials is no assurance that children are learning more. If children merely manipulate materials without proper guidance and direction, little insight into what is being attempted may result. The teacher must know the possibilities, the limitations, and the correct use of each kind of instructional aid. With proper background and understanding the teacher can teach children how to use different materials so that true quantitative thinking evolves.

Instructional aids enable the teacher of these fifth grade children to provide for individual needs. (Vincennes Public Schools)

A meaningful mathematics program places emphasis on the process of learning. This means that each kind of instructional aid must function properly. As children grow in their ability to handle number ideas they should be guided toward the use of semi-concrete materials and of symbolic materials. It should be constantly kept in mind that different children need different levels of experiences with different types and levels of instructional aids. Great claims have been made for one type or another of instructional aids. It is difficult to envision only one type of material meeting all the needs and abilities of all the children in one particular grade. Individual experimentation and discovery by individual children is the heart of any modern mathematics program.

Concrete and Semi-Concrete Aids in Grade 2*

When presenting new number facts, each child is given a 6" x 9" sheet of colored construction paper. This sheet is divided in half by a white crayola line. Each child is also given a small envelope containing 10 cardboard disks.

The children choose a number from 1 to 10. For example, if they choose 7 as a number, each child takes out 7 disks from his envelope. The children arrange this total group into two sub-groups, dividing the 7 disks into any size groups they wish. One of the sub-groups is represented on one half the paper and the other sub-group on the other half of the paper. The disks are arranged so that the children can recognize the groups without counting. As the different children report their groupings, the teacher records them on a flannel board in the front of the room and places the symbolic number facts on the chalkboard.

The children are then instructed to slide their sheets around. This of course reverses the order of the sub-groups. Different children are asked to tell the new number stories to the recorder who translates them onto the flannel board and the chalkboard alongside the previously recorded and related facts.

After the children have developed number facts related to such numbers as 5, 6, and 7, the number sheets and disks are replaced with pieces of Oaktag 6" square, with which the children make their own number cards. These cards are folded in half. On the right-hand side the children use black dots to represent the symbolic number fact recorded on the left-hand side. Thus, the symbolic representation is recorded along with the semi-concrete diagrammatic representations. A

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*Adapted from a practice submitted by Marion A. Clark, Vigo School, Vincennes.

number card is made for each fact discovered that day. Later the children cut the cards apart and use them for individual and small group practice and games.

Concrete and semi-concrete instructional aids help these second grade children understand and learn addition facts with sums of seven. (Hammond Public Schools)

Generalizations

1. A basic fact in addition is the combination of any two one-place numbers and the answer.

2. Addition is a regrouping process.

3. Each large group is composed of several different but related combinations of sub-groups.

4. Interchanging the sub-groups in a number fact does not change the value of the total group. (Commutative law)

5. Each child should have materials provided which will help him to make discoveries of relationships among groups and sub-groups.

6. Symbolic representations should be written records of meaningful experiences.

7. Children should have many opportunities to practice using number facts in a variety of ways.

Concrete and Semi-Concrete Aids in Grade 3*

For a class which had demonstrated a high level of readiness for division, a lesson was designed to clarify and refine division concepts. Although the children had not used the division algorism, they could verbalize many basic division facts.

Each child was given a 10-bead counter (a single wire with 10 plastic beads). The children counted the beads to establish the size of the total group. Then they proceeded to try to divide the total group into two groups, five groups, three groups. When they attempted to divide the 10 beads into three groups with the same number of beads in each group, several exclamations were heard. "It won't work." "There's one left over." There was a brief discussion of the children's discoveries related to the sub-groups of 10.

Each child was then given a sheet of paper. On each sheet was drawn a picture of an open candy box containing 12 compartments. There was a piece of candy pictured in each compartment. Each child was given 12 counters and asked to place them in each compartment of the box, pretending that each counter was a piece of candy. The class was asked to find out all the different ways they could empty the box, taking out the same number of pieces of candy each time. The following examples of number combinations evolved:

1 removed at a time 12 times
2 removed at a time 6 times
3 removed at a time 4 times

All of the possible combinations discovered by the class were recorded on a chart by the teacher.

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*Adapted from a practice submitted by Ronna Riddle, School No. 55, Indianapolis.

Generalizations

1. Division involves regrouping.

2. Division problems can be solved by counting.

3. Of a given group, the larger the number of equal sub-groups the smaller the size of the sub-groups.

4. Of a given group, the smaller the number of equal sub-groups the larger the size of the sub-groups.

5. Division can be interpreted as a shortened form of subtraction.

6. The language of division involves the use of the word "times."

PROMOTING PROBLEM-SOLVING ABILITY

Confusion and anxiety seem to be the by-products when problem-solving programs are discussed. From such discussions it would seem that problem solving is only a part of the mathematics program. Such a compartmentalization of mathematics is unrealistic. Actually, problem solving is at the heart of and permeates all of every modern mathematics program. It should be conceived of not as an isolated skill but rather as the highest form of reflective thinking which grows out of every facet of mathematics.

Various approaches to the teaching of skill in problem solving have been used, some of the more familiar of which are formal analysis, formal steps or procedures, and "cue" words. Significant research dealing with each of these approaches has raised serious doubts as to its effectiveness.

Problem solving assumes new dimensions for these sixth grade children when they use a variety of instructional aids to help visualize the quantitative situations. (Vincennes Public Schools)

Attention to vocabulary control and reading skills related to verbal problems has also failed to be the panacea for developing high level problem-solving abilities. More and more mathematicians are recognizing the complex nature of the skills needed to promote effective problem solving in mathematics. They are also becoming more aware of the fact that different children use different approaches in dealing with the quantitative problems they need to solve.

Real supplies, real money help these third graders to have many "real" problem-solving experiences and provide a real purpose for practicing computation in mathematics. (Vincennes Public Schools)

Every teacher should place emphasis upon effective quantitative thinking as the basis for success in problem solving. Such thinking takes place when children are personally involved with quantitative situations that cannot be handled in a habitual way. Using experimentation and exploration as well as what they already know about number facts and relationships, children should work toward tentative plans for the solution of their selected problems. Evaluation and organiza-

tion of the available data and plans allow the children to select a variety of possible operations that can be used. Under the guidance of the teacher they then further evaluate the plans, operations, and solutions to their problems. Each conclusion is tested against the original problem to ascertain its effectiveness. It is important that several plans, operations, and conclusions be presented.

It would seem to be an obvious truism that children learn to solve problems by having many opportunities to solve problems. Yet some programs still present computation procedures first and then present the placement of these processes in a verbal context. Such an instructional program leaves out many important opportunities to relate problem solving to the genuine concerns and interests of children. The school environment produces many concrete situations every day which provide excellent opportunities for real experiences in quantitative thinking. Children's experiences outside of school also suggest many situations which can become an effective part of the problem-solving program. Illustrative verbal problems can be used to introduce a new number fact or process and can be used effectively for review and/or practice. They can provide variety and significance to the procedures being studied.

Additional opportunities to solve quantitative problems will help, but the mere solution of problems does not necessarily develop understanding or efficiency. A modern problem-solving program in mathematics recognizes the need for time to think, explore, experiment, evaluate, and discuss. Thus quantitative thinking evolves.

Problem Solving in Grade 5*

The class was divided into three groups, and each group was given a set of pictures. Group I was asked to make up written number problems dealing with multiplication; group II, problems dealing with division; and group III, problems dealing with fraction concepts. Each group used the pictures as ideas for the setting of their problems.

After each child had written at least one problem, the problems were given to an editorial committee chosen by the children. This committee examined each problem in light of the following criteria:

1. Was the problem well-written?
2. Was the problem interesting?
3. Was the question to be answered clearly stated?

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*Adapted from practices submitted by Norma Brunt, School No. 3, Indianapolis; Myrtle Day, Washington School, Hammond; Fannie Lloyd, Georgetown School, New Albany; and Ann S. Niemann, Washington School, Vincennes.

After the committee had selected what it felt were the best problems, these were submitted to the class for solutions.

Each child was asked to solve each of several selected problems, and was encouraged to draw pictures or diagrams of problems and their solutions. The numerical solutions and diagrams were submitted to another committee of children who selected the solutions and the "best" diagrams for each problem as well as what it considered to be a different but acceptable solution for each problem. A diagrammatic solution and two numerical solutions for each problem were put into a booklet entitled Our Own Problems in Mathematics.

Generalizations

1. The statement of a process in a verbal context helps demonstrate a child's understanding of that process.

2. Verbal problems can be solved in several different ways.

3. Problems can be better understood when certain criteria are applied to children's own quantitative statements.

4. The language of mathematics must be clear and precise.

5. The elements of a verbal number problem are better understood when children have opportunities to compare similar elements themselves.

6. Mathematical problems are excellent vehicles for teaching skills in the language arts.

Problem Solving in Grade 6*

In planning a class trip to the Art Institute of Chicago, the children are presented with the problem of deciding how much money each child must contribute to finance the trip. One or two children serve as moderators and recorders (they call on the others for suggestions and also write all pertinent data and arithmetic processes on the blackboard where it can be reviewed and studied by the entire class). First the children choose between a $25.00 bus rental and a $30.00 bus rental. The $30.00 rental will give them an additional hour at the museum. They estimate the time for travel, time for lunch, time for restrooms, and time for visiting the galleries. Discussion of

*Adapted from a practice submitted by Lynn Olson, Lafayette School, Hammond.

the estimates serves to clarify the problem. The $30.00 bus rental is chosen because the additional time is needed. The discussion also points up the need to add to the $30.00 an estimate of the money required for phone calls to the museum and to the bus company. The children now determine how much money each child will need to contribute. They divide the total expense by the number of pupils in the room.

The pupils now plan a schedule for the use of their time at the art museum. The teacher has prepared a map of the three floors of the museum and has keyed this map with an index of the exhibits, the cafeteria, coat room, museum store, and restrooms. The map and the index are mimeographed and distributed to the children. The children use books, illustrations, and reproductions to decide which exhibits they want to see. They then proceed to design a time schedule in the form of a chart showing the place, time of arrival, time spent, and time of departure for each aspect of the trip including travel time, cafeteria time, restroom time, gallery or exhibit visiting time, and time to purchase souvenirs in the museum store. They learn how to orient the map. They carry the map and the time schedule to the museum and use both to guide themselves about the museum instead of being led from one hall to another by a teacher or a guide.

The children also use road maps and decide upon a route of travel in the bus to and from the museum. They decide whether to use the skyway (a toll road), or the expressway (a freeway). Mileage is considered and cost per mile is figured. The children carry the maps with them on the bus and locate landmarks, roads, and points of interest. They orient the road maps on the bus and determine direction of travel at various times.

Generalizations

1. Classroom activities provide many opportunities for solving "real" problems.

2. Quantitative situations which grow out of actual experiences are significant to children.

3. Group discussion of quantitative situations provides excellent practice in problem solving.

4. Estimation is an essential part of problem-solving procedures.

5. When solving "real" problems computation is only a means by which significant information is obtained.

6. Significant problems involve both quantitative and qualitative judgments.

PROMOTING QUANTITATIVE VOCABULARY AND READING SKILLS

Mathematics has a vocabulary all its own. It also demands specific, specialized reading skills which differ significantly from the general reading skills used with literary materials. A modern mathematics program should place considerable emphasis upon the development of quantitative vocabulary and related reading skills.

In developing a meaningful vocabulary in mathematics, attention must be given to two types of quantitative terminology--definite and indefinite. Definite terms deal with such concepts as acre, inch, and dollar. Indefinite terms deal with such concepts as big, little, and more. With both definite and indefinite terms, the teacher needs to help children build meaningful frames of reference. Exact frames of reference can be developed for definite terms. An acre can be measured, walked around, and visualized. A mile can be designated from one known point to another. Indefinite terms, however, must be established on relative bases. Big is always determined in relation to something else which is either bigger or smaller. Children need many experiences which will enable them to expand their concepts of both definite and indefinite quantitative terms. Mere dictionary definitions are not enough.

In addition to definite and indefinite quantitative terms, teachers need to concern themselves with the technical vocabulary of mathematics. Such words as numerator, divisor, and hypotenuse become meaningful as children have experiences with them in a variety of meaningful contexts. Certain words in mathematics have both a literary and a mathematical meaning. Children should have opportunities to distinguish and understand the various meanings and to know where they best apply. The meaning of dividend from a purchased stock is very different from dividend as part of the division process. A recognition of the job to be done in developing quantitative vocabulary is essential for the development of any mathematics program, and the provision of many experiences wherein quantitative terms can be meaningfully developed is essential if a modern mathematics program is to meet its objectives successfully.

The development of a meaningful quantitative vocabulary is only one step in the teaching of the special skills needed for reading in mathematics. A wide range of other special skills should also be given specific attention. Children need to be able to read numerical symbols, symbols of operation, notation, and algorisms and to comprehend their meaning. They need certain basic skills in reading verbal problems. This involves far more than just a general comprehension--it calls for the development of skill in specific, analytical comprehension, in determining relationships and procedures, and in locating information not stated in the context of the reading itself.

Reading recipes and following the quantitative directions requires these first grade children to use specific mathematical skills. (New Albany-Floyd County Consolidated Schools)

Mathematics makes use of many quantitative aids and measuring devices which involve specific reading skills. Clocks, yardsticks, calendars, statistical tables, graphs, charts, maps are just a few of these aids and devices which need attention in the development of specialized reading skills in our reading program in mathematics. Provision must be made for teaching all of the specific, specialized reading skills within the regular mathematics program. The general reading program cannot be relied upon to do the job for us.

Quantitative Vocabulary in Grade 3*

At the beginning of the year the children started planning for the development of their own mathematics center. They decided they would need a mathematics dictionary to help them with

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*Adapted from practices submitted by Charlotte Fertig, Lydia Middleton School, Madison; Janet Steger, School No. 86, Indianapolis; and Virginia Moore, School No. 55, Indianapolis.

new mathematical terms but, as they collected the needed materials, they found that they were unable to locate a mathematics dictionary. After considerable discussion the children decided to make their own dictionary.

At first they had difficulty deciding the size they would need. It was suggested that the new words and information about them could be put on a large chart. It soon was evident that a 12" x 18" dictionary would be needed. This was to be loose-leaf so that new words could be added as the children discovered them.

As the dictionary took shape, the children eagerly sought new mathematical terms to be included in it and worked out definitions to be given with each word. These definitions came from the context of lessons or of the children's reading. Although the standard dictionary was consulted for additional help, the definitions were stated in the children's own language. The fact that a word had been entered previously did not mean it had been forgotten. On the contrary, the children were eager to enter additional information about "old friends." Wherever possible, diagrams and pictures were included to help clarify meanings of terms

The mathematics dictionary was in constant use throughout the year either to enter new words, to list additional meanings to words already included, or to help check understandings and meanings of quantitative terms being used every day.

Generalizations

1. Children are interested in quantitative terms when they are allowed to discover these terms and develop their own definitions out of meaningful experiences.

2. As children gain in their insights into mathematics, quantitative terms take on broader and additional meanings.

3. Children remember best the quantitative terms and meanings that are couched in their own language.

4. Children need a ready resource to check their understanding and spelling of quantitative terms.

5. A child's quantitative vocabulary is expanded in proportion to the number of opportunities he has to use that vocabulary in meaningful ways.

Reading Skills in Grade 5*

Reading in mathematics is not just a lesson or two but a whole year's project. During the year the children engage in a variety of activities which will extend their reading skills in mathematics.

Time is set aside each week for reading, interpreting, and analyzing verbal problems. During this time little or no attention is given to finding answers. The children occasionally work as a total group, but most of the time smaller groups work with specially prepared problems geared to their ability level. Problems are read silently, and then the children are asked to state the problems in their own words. The children's interpretations often clearly indicate lack of or erroneous understanding of quantitative words, phrases, and symbols. When this is the case, time is taken to develop or clarify concepts related to quantitative terminology. Individual notebooks are developed and new words and their meanings are included in one section entitled Important Words in Mathematics. In another section, devoted to Important Signs in Mathematics, diagrams and examples using the signs are placed.

Occasionally, the children interpret their reading of verbal problems by dramatizing the situation. Real people and concrete objects often make the verbal problem "come alive." Such dramatization helps to clarify quantitative terms and symbols that otherwise might remain "fuzzy" or misunderstood.

Considerable attention is devoted to the reading of directions to be used in the work in mathematics. The class never proceeds in actual computation or problem solving until it is clear that every child understands what is to be done. The group or groups always do some work together, following the directions given. Specific attention is devoted to the elements contained in the directions. Words, phrases, and signs are carefully clarified before individuals proceed in their work.

Whenever a new aspect of the work is to be introduced, large vocabulary charts are prepared on 18" x 36" paper. The charts include all new quantitative words and symbols found in the new work. These charts are posted where they can be seen by all the children, and they remain in the room until the children demonstrate that they are thoroughly familiar with the terms. To facilitate the acquisition of the new vocabulary, two sets of cards are made, with the new word or phrase written on one set and a definition of the word or phrase on the other.

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*Adapted from practices submitted by Lois Arthur, Elm Heights School, Bloomington; Darlene Goudy, Riley School, Vincennes; and Jack Hayes, School No. 55, Indianapolis.

These cards provide small groups of children with many hours of practice and enjoyment when they are used as a vocabulary game.

Generalizations

1. Reading in mathematics is a specialized kind of reading.

2. Specific reading skills need to be taught during the mathematics period.

3. Written directions and verbal problems offer many opportunities to promote more effective reading skills in mathematics.

4. Children's interpretations and dramatizations of quantitative situations are valuable learning experiences for teaching reading in mathematics.

5. Instructional aids such as concrete objects, wall charts, notebooks, and games facilitate the acquisition of quantitative vocabulary.

MAKING, USING, AND INTERPRETING GRAPHS

Graphic representation of tabular data is being used more and more in all media of mass communication. Intelligent interpretation of the number relationships presented graphically in newspapers, magazines, books, and television programs must be considered an essential objective of a modern mathematics program.

A circle graph helps these fifth graders interpret data related to the school safety program. (Hammond Public Schools)

Specific skills are involved in making, using, and interpreting graphs. These skills must be taught. A modern program does not wait until the intermediate or upper grades to begin developing these skills; it starts work with graphs in the kindergarten. With help from the teacher, kindergarten children can make simple bar graphs from

recorded temperatures during a week's time, or graph the number of children present each day during a week. From such experiences they begin to understand some of the mechanical features of a graph. When they compare the temperature on different days or the attendance on Monday with that on Wednesday, they are learning to interpret data. These skills become invaluable as they encounter graphic representations in their other subjects and reading materials.

Later in their school experiences children learn about a variety of kinds of graphs: line, circle, pictorial. The interpretation of the data from these various graphs depends in large measure upon the richness of background of the learners. As the children develop their own graphic representations they should begin to realize that there are times when data should be reported precisely and other times when close approximations, using round numbers, is sufficient. Graphs are usually developed from a table of information. Children must have opportunities to understand and apply the relationships which exist between tables and graphs. They need opportunities to grow in their abilities to handle both factual interpretations of graphic data and inferential interpretations of graphic data.

Hardly a day goes by without some experience in the elementary school that lends itself to the making, using, or interpreting of quantitative information graphically. A modern mathematics program seizes upon these real experiences to promote children's skills in graphic representation.

Graphs in Kindergarten*

For several weeks during February the teacher kept a number chart for each week's attendance. Each morning, the children counted those present and the teacher recorded the number on the chart opposite the day of the week. The days were arranged in vertical form.

One Monday the children's attention was called to a new chart. The days of the week were arranged just as they were on the old charts, but now this chart was much bigger and a line had been drawn along the bottom. Each child was asked to go to the chart and to paste a one-inch square of black construction paper opposite the word "Monday." The squares were counted and the number recorded on the base line. Then all the children present were counted and the number was compared with that on the chart. The numbers were found to be the same. The children did this each day for a week. On Friday, the chart was used to answer such questions as: On what day

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*Adapted from practices observed in University School, Bloomington; and from practices submitted by Ann Niemann, Washington School, Vincennes.

were the most children present? On what day were the fewest children present? Which was easier to read to help answer our questions, the picture chart (bar graph) or the number chart? Why? The children enjoyed making the picture chart so much that they decided to make another for the following week.

Generalizations

1. Kindergarten children can make and read graphs.

2. A graph is easier to read and use than a table of numbers.

3. The length of each bar on a bar graph is made up of equal units.

4. All bars on a bar graph use the same sized unit.

5. Graphs provide opportunities to use both definite and indefinite quantitative terms in a meaningful way.

6. A graph helps in making comparisons of data.

7. There is more than one way to record information.

Graphs in Grades 5 and 6*

In the fifth and sixth grades, work with graphs is carried on throughout the year. The children make, use, and interpret pictographs, line graphs, bar graphs, and circle graphs. In order to gain facility in the use and interpretation of graphic representations, a variety of activities are carried out.

During the first part of the year the children search for and bring in different types of graphic representations from magazines, newspapers, places of business, books, etc. Attractive bulletin board displays, charts, and a reference book of graphs are prepared by the children. Different types of graphs are interpreted and discussed, and the advantages and disadvantages of different graphic representations are presented to the group.

After the initial work with representative types of graphs, the children begin to make their own graphs. Using previous discussion, the bulletin board, and the resource book of graphs as references, they prepare graphs which include

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*Adapted from practices submitted by Catherine Davis, School No. 53, Indianapolis; Helen Halley, School No. 15, Indianapolis; Luan Smith, Hillside School, Bloomington; and Anna D. Weiland, Caldwell School, Hammond.

data about their daily lives. Some extremely unique ones have been made concerning their visits to relatives, allowances, time spent doing chores at home, time spent in sleep, etc.

The children next prepare graphs for use by the school officials concerning such things as attendance records, enrollments, supplies, lunchroom statistics, etc. These graphs are displayed in the principal's office.

Attention to graphs in the mathematics program helps the children become especially sensitive to the importance of graphic representations in their social studies and science work. Whenever reports are given to the class, graphs are sure to be used in presenting quantitative information. Whenever graphs are located in the social studies and science textbooks, time is devoted to the interpretation and discussion of the graphic representation.

In their mathematics notebooks, the children keep individual achievement graphs. These graphs, either line or bar representations, are kept on a weekly basis. Each Friday time is allotted to recording that week's achievements. Thus no week goes by without some attention being devoted to making, using, and interpreting graphs.

Generalizations

1. There are many kinds of information that can be represented graphically.

2. Certain types of graphs are best suited for the presentation of particular kinds of data.

3. Some types of graphs show number relationships among quantities much more clearly than do other types of graphs.

4. Mass media rely heavily upon graphic representation of quantitative data.

5. Familiar topics provide meaningful data for the development of graphic representations by the children.

6. The reading of graphs and the interpretation of the data are essential to the discovery of quantitative relationships in all areas of the curriculum.

TEACHING HISTORY AND DEVELOPMENT OF OUR NUMBER SYSTEM

Too long has the history and development of our number system been omitted from our programs. We need to draw upon the past achievements and systematic evolution of mathematics to understand more clearly our present status and future developments. A modern mathematics program enables teachers and children to tap the reservoir of history to better accomplish their mathematical goals.

Where meaning and understanding are of prime importance, and interest and positive attitudes are desired outcomes, we need a historical perspective. Broad objectives, such as the following, can be better accomplished as children study this important aspect of the science of mathematics:

1. To develop an appreciation of the Hindu-Arabic number system as compared with other number systems.

2. To develop an understanding of the relationships and properties of the decimal number system.

3. To develop an understanding of the dependence of civilization upon adequate mathematics.

4. To develop a rich and meaningful quantitative vocabulary.

The simplicity and efficiency of many of the abstract number concepts related to our number system stand out in "bold relief" when compared with those of the Egyptian or Roman systems of notation. As we probe the roots of antiquity and follow the evolution of our number system, we gain a deeper insight into the importance of and need for the base of a number system.

Simple comparison of our number symbols 0 - 9 with the complex notational system of other cultures helps to build an understanding of the properties and wonders of 0 - 9. Experiences with number bases other than 10 give children a better understanding and a greater appreciation of the concept that in our number system the value of every digit in a positional notation depends upon its position.

A study of the history and development of our number system reveals that the very existence of all civilization has depended upon mathematics. Teachers have the opportunity to help their children realize that modern mathematics is a creation of all nations and that the language of mathematics is truly international.

The rich and colorful history of mathematics provides us with countless opportunities to build a meaningful quantitative vocabulary with children. Many of the words we use every day had their origin in some historical time or incident. Words such as "counter,"

"bankrupt," "tally," and "stock" can be traced to medieval England and the life of that time. Such glimpses into the past add depth and richness to the meaning of quantitative terms.

Sixth grade children check accuracy of early units of measurement in their study of the history and development of measurement. (Hammond Public Schools)

The history and development of measurement is a fascinating story to children. Some of the most ancient methods of measuring are

still in use today. The first measures of length were associated with different convenient parts of the body. The first measures of capacity were related to man's possessions or to himself. Opportunities to use history to discover these origins of measurement allow children to better understand that one of the fundamental ideas of measurement is that of standardized substitution.

Any teacher interested in improving the teaching of mathematics must help children weave the history of mathematics into the tapestry of their number experiences.

History of the Number System in Grade 4*

Each week during the year the teacher reads number stories to the class. Sometimes these stories, both factual and fictional, are historical in nature; at other times they are related to present-day situations.

Often these stories act as a base for the development of short-term activities related to the mathematics program. One such activity grew out of the reading of David E. Smith's Number Stories of Long Ago.

In the discussion following the reading of the stories, the children learned that the stories were mostly fiction but that they contained some facts about the history of our number system. It was decided that the children would attempt to write some number stories of their own. In order to do this, they needed to do some research to ascertain certain historical facts around which they could weave their number stories. This meant turning to such resources as the encyclopedia and the Rowe Peterson Enrichment Series.

Each child chose some historical information about the beginnings of our number system. The criteria used for the stories were that the historical number facts should be true and the setting or plot could be true. The number stories were read to the class and criticized on the basis of originality, interest, factual information, and setting. The stories were then bound into a booklet called The Beginning of Our Number System. This booklet was shared with other classes in the school. An example of one of the stories follows:

"Notok was a mighty hunter. He traveled great distances hunting animals. He could not bring all the animals he killed back to show his friends. He wanted to tell his friends about

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*Related practices submitted by Mary Harris, McKinley School, Columbus; and Delma Beckhort, East Spring Street School, New Albany.

the animals he killed. On his next trip he decided to bring back a bone from each animal he killed. He killed so many animals that the bones became very heavy to carry. He decided to exchange each bone for a small stick. These were easy to carry and now he could show his friends how many animals he had killed.

--By Jerry Lester, 4th grade"

Generalizations

1. The beginnings of our number system are lost in the roots of antiquity.

2. Primitive man used one-to-one matching in dealing with quantities.

3. Early in man's history he needed to find answers to such questions as "how much" and "how many."

4. There is a great difference between our present methods of recording numbers and early man's methods.

5. It has taken man thousands of years to create our present number system.

USING INSTRUCTIONAL AIDS CENTERS AND BULLETIN BOARDS

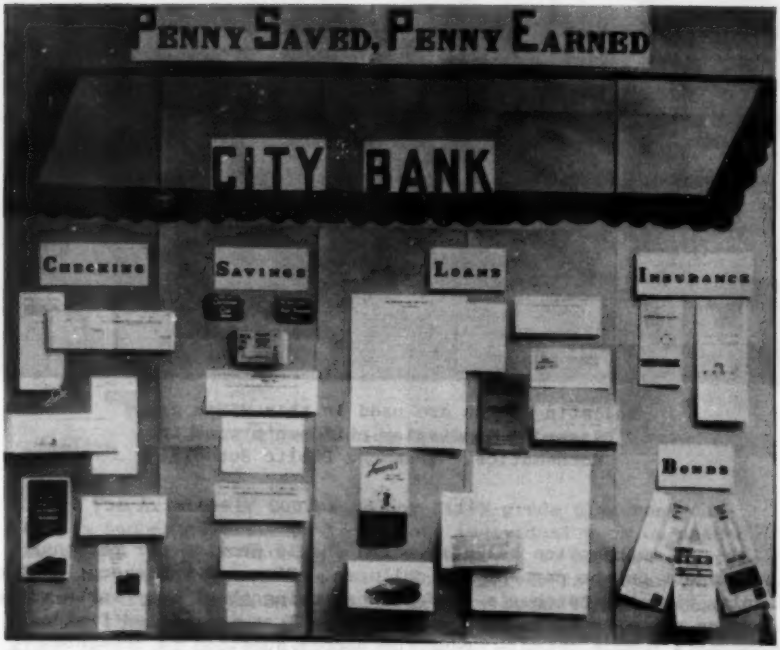
The classroom should be a learning laboratory if it is to be in line with what we know about how children learn. All laboratories need a place where the materials related to desired learnings may be displayed, experimented with, and developed. Mathematics is no exception.

An instructional aids center provides these primary children with many opportunities to experiment with manipulative learning devices.
(Hammond Public Schools)

A mathematics instructional aids center provides opportunities for individuals or small groups to pursue number activities during their free time or during the mathematics period. It allows the teacher to differentiate assignments and to provide appropriate materials for different levels of learning. When different instructional

materials are needed at the children's desks, the center can serve as a materials cafeteria. The center may contain the following instructional aids at the appropriate grade levels: (1) counters (tongue depressors, 10 and 20 bead counters, knotted strings); (2) hundreds number squares and peg boards; (3) fact finders; (4) flash cards; (5) measuring devices of all types; (6) various abaci (simplified abacus, Japanese abacus, Chinese abacus); (7) number games; (8) fraction boards; (9) sets of geometric shapes; (10) mail order catalogues and consumer guides.

These are but a few of the instructional aids which children of different age levels may use for exploration, experimentation, discovery, and practice in working with number facts and number relationships. A modern mathematics program would indeed suffer without the use of an instructional aids center as part of its learning laboratory.



An "informational type" bulletin board reflects a fifth grade's study of banks and their functions. (Madison Consolidated Schools)

The mathematics bulletin board is a genuine asset to the learning environment of the modern mathematics program. It can be used to

provide background information of a topic being studied or to display unique and challenging number situations, thought-provoking number riddles, magic squares, and cross number puzzles.

Bulletin boards are used in this sixth grade classroom to display children's work in mathematics. (Hammond Public Schools)

The mathematics bulletin board should provide for personal involvement on the part of the children. Planning and executing the display with the teacher's guidance can bring about many worth-while learning experiences in mathematics as well as in related fields. Whenever a puzzle, riddle, game, or number question is presented, the children should be given an opportunity to respond to the questions or problems displayed. Bulletin boards, properly used, can make significant contributions to the concept of the classroom as a learning laboratory in mathematics.

An Instructional Aids Center in Grade 1*

The Arithmetic Center in our first grade classroom is on a table in an area backed by a bulletin board. The arrangement is permanent, except for addition of materials as more are needed or discovered.

The bulletin board holds the name of the area, lettered in four-inch manuscript with 3D shadow emphasis. Here are placed flat materials, such as the calendar, the mock-up thermometer with sliding tape for mercury, the model clock with movable hands, and charts which show a numeral, its name, and a diagram using that number of objects.

On the table are peg boards and colored pegs, blocks, circles, number books, number puzzles, an egg carton with wooden eggs, plastic pint and quart bottles, measuring cups, measuring spoons, a plate with wooden fruit in fractional parts, a ruler yard stick, a tape measure, toy money, a toy telephone, and a real thermometer.

Also available are individual felt boards with felt numbers and objects, including rabbits, ducks, birds, apples, pears, circles, and squares.

The hundred board, a very special one, is hung nearby.

The children are free to experiment with these objects at their leisure. They are used to illustrate problems from the chalkboard or original problems which the children suggest.

The Center makes the pupils more conscious of the uses of arithmetic and its importance in our daily lives.

An Instructional Aids Center in Grade 5**

The Arithmetic Center in our fifth grade classroom contains a number of articles to promote individual work on needed skills and "mind-stretching" and incentives to do just a little more than is required. The table which serves as the arithmetic center contains a number of supplementary books and textbooks, most of them are fifth grade books, but a few are at the fourth grade level and one or two are at the third grade level. Nearly all these books are checked out at all times. When a pupil

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*Practice submitted by Freda M. Dysert, School No. 55, Indianapolis.

**Practice submitted by Anne Miller, Fairmont School, New Albany.

feels he needs extra work on a concept or finds something he particularly enjoys and wants to do for extra credit, he takes one of the books, chooses a page or pages, and hands in his completed work for checking. One feature enjoyed especially by the girls is an old, old manual containing problems, examples, and answers. The children use this book to test one another during the noon hour, taking turns working the problems on the board and checking them.

A box containing counting discs, clothespins, and other concrete devices has been pretty well neglected but is available if needed. There is also a box of play money for practice in change-making.

Among the commercial games on our table are two sets of "Quizmo" for the four basic processes; sets of flashcards for all the processes; "Puzzle Plans" for some processes; "Fractions Can Be Fun;" the "Parking Lot" multiplication game; and--a special favorite--an electrical combination board which lights up when the correct answer is chosen.

Teacher-prepared games and aids include various flashcard sets, one with measurement abbreviations and equivalents, another with arithmetic vocabulary, and others with simple problems.

The flannelboard has been very useful in basic fraction work, using commercial and teacher-prepared fraction equivalents. Flannel cutouts of measuring devices have been used with special enjoyment when children bring recipes from home and demonstrate with flannel cutouts just how much of each ingredient is needed.

Large Roman numerals, teacher-made of white flannel, allow for practice in forming those fascinating Roman clock faces and other numbers. There are also flannelboard cards of the Roman and Arabic equivalents for matching.

A cardboard clock face with movable hands has been a help to some fifth graders who are still shaky about time-telling.

A four-foot-high electrical matching board, which can be re-programmed for each change of questions to prevent memorization of the correct electrical outlets, is used for many types of questions--measures, fractions, facts, vocabulary, etc.

Along with the arithmetic table, the class has done some work on bulletin boards that shows the practical uses of arithmetic (this class has been particularly scornful of thought problems which are "unrealistic"). The current display, entitled "We Use Fractions Every Day," shows pupil drawings of the way they may use fractions. Many of the boys, of course, have depicted sports events played in "halves" and "quarters." A number of girls have concentrated on cooking and measuring devices.

Others have thought of time, money, distance, volume, and progress. A carpenter's son drew a detailed scene showing builders at work on a house, labeled "2/3 done by noon." It was gratifying to see how aware these children have become of the reason for learning fractions and for the use of fractions--and arithmetic--in their daily lives.

Generalizations

1. Each classroom, regardless of grade level, needs an instructional aids center.

2. A variety of manipulative and exploratory instructional aids helps to make number and number operations meaningful to children.

3. An instructional aids center can help to individualize the mathematics program.

4. An instructional aids center can provide activities for leisure-time pursuits.

5. An instructional aids center can offer challenging enrichment activities.

6. The instructional aids center can provide interesting practice materials and activities.

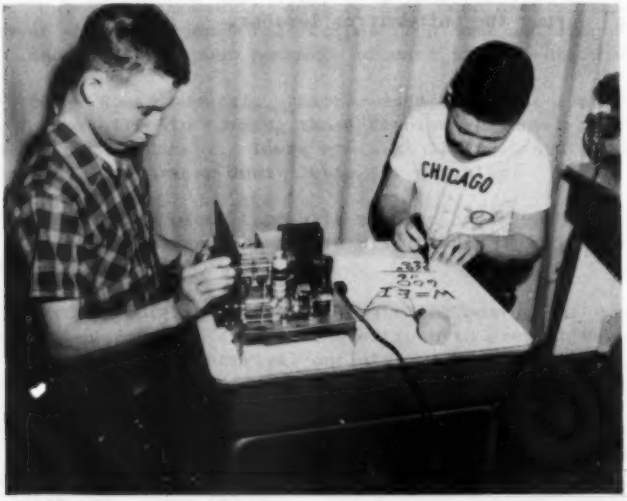
7. An instructional aids center can be used as a "materials cafeteria" during the mathematics lessons.

INTEGRATING MATHEMATICS WITH THE OTHER SUBJECT AREAS

Children need to be sensitized to the quantitative problems which exist in other fields. The mathematical demands in these areas will contribute markedly to increased motivation and interest in mathematics.

The amount of mathematics that is learned in the study of other fields depends upon recognition of the mathematical elements involved, adequate treatment of the mathematical concepts, and proper application of the quantitative understandings to the context within which the mathematical elements are needed.

The heavy incidence of quantitative terms and ideas in social studies and science are indicative of the distinctive contributions which these fields can make to the modern mathematics program. Yet it should be pointed out that this is by no means a one-way street. The programs in both social studies and science profit in direct proportion to the quantitative understandings children bring to their study of these subjects.



These sixth grade pupils demonstrate the close relationships which exist between electronics and mathematics.
(New Albany-Floyd County Consolidated Schools)

Art experiences provide many opportunities for the use of measurement and of geometric shapes and designs. Quantitative terminology permeates reading books and supplementary resource books. Health, safety, physical education, in fact every field makes its demands upon mathematics and contributes valuable, real settings for true mathematical experiences.

We are constantly being challenged to use real problems in our mathematics programs. Every other field in the school program presents us with a multitude of opportunities to select such problems from needed and meaningful contexts. A modern mathematics program accepts this challenge.

Integration with Science in Grades 2 and 3*

Along with a science unit on weather, the children learn to read the thermometer. They keep a weather calendar using a sun, cloud, snowman, umbrella, etc., which the children make, to indicate the weather for the different days.

The weather reporter for the day takes the thermometer outside when he arrives at school. Just before recess he goes outside and reads the temperature. When he returns to the room, he fixes the large toy thermometer so that the class may read the temperature. The temperature is recorded on the weather calendar.

Questions such as the following may be asked: Is the temperature higher or lower than yesterday? How many degrees higher or lower? What has been the highest temperature for the week or month? the lowest?

Large bar graphs are made of the temperature recordings for each week. Five-degree intervals are used to give additional practice in counting by fives as well as to help the children see one of the practical uses of counting this way.

As the days go by other questions using the arithmetic vocabulary may be asked: How many sunny days have we had? How many cloudy days? How many more sunny days than cloudy days? Have we had fewer (or more) rainy days than cloudy days?

Number stories using the weather information may be told to the children and someone may be asked to work the problem on the board for all to see. Later on some children are able to write their own number stories. For example:

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*Adapted from practices submitted by Mary Pastor, Morton School, Hammond; Joan Collins, Rogers School, Bloomington; Juanita Cauble, Green Valley School, New Albany.

A daily recording on their weather chart provides these fourth grade children many opportunities to use mathematics in their study of science. (New Albany-Floyd County Consolidated Schools)

"Yesterday it was 46°. Today it is 32°. How many degrees higher was it yesterday?"

"We have had 9 sunny days and 4 cloudy days. How many more sunny days than cloudy days have we had?"

At the end of the month a summary is made of the weather, including the number of sunny days, number of cloudy days, number of partly cloudy days, number of snowy days, number of rainy days, highest temperature, and lowest temperature.

If the children are still interested, a weather calendar is kept for the following month; comparisons are made of the two months.

Integration with Science in the Primary Grades*

A flower garden project at an underprivileged school afforded an opportunity for children in the primary grades to use

*A practice submitted by Claire Haywood, School No. 53, Indianapolis.

a concrete situation for better understanding of certain mathematics concepts. The project was initiated in the spring as an outgrowth of a series of science lessons on plant life. A school flower garden grew out of the planning. This was a long-range project involving planting in the spring, cultivation and care all summer, and gathering and distributing the flowers, and later the seeds, in the fall.

Garden plans were studied, seed purchases made, flower arrangements displayed, and dozens of packages of new seeds distributed during open house in the fall.

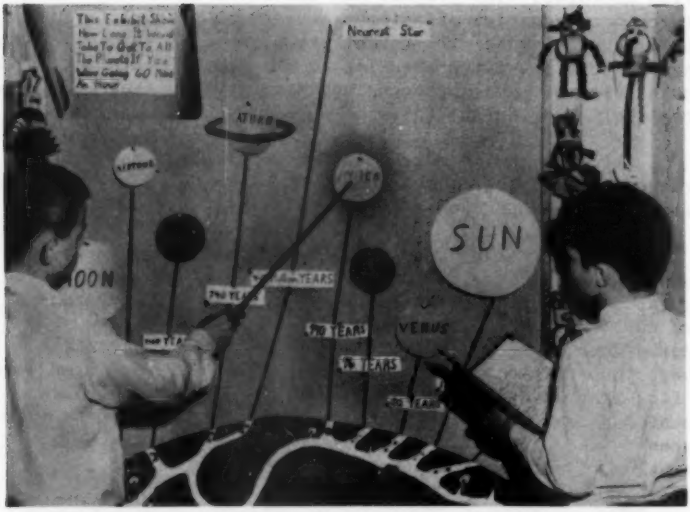
There were many concrete situations for understanding "inch," "foot," "yard," "length," and "width" as they plotted the garden. They compared costs and quantities. Records were kept of the growing period. Packages were counted and grouped into dozens. Such terms as "tall, taller, tallest," "short, shorter, shortest," "more, less" and "fewer" were used as flower arrangements were discussed. Many of the colored spots of paper beneath the arrangements were cut in geometric shapes by the children. There was much learning which gave attention to mathematical concepts.

Integration with Science in Grade 6*

When the class was studying soil and soil drainage, one boy brought a half can of garden fertilizer (chemical) to school. The children divided the full net weight that could be put in the can by two to estimate the amount of fertilizer they had in the can--one and one-half pounds divided by two.

The children designed a chart to organize a test of the fertilizer and to determine the optimum ratio of fertilizer to a given type of soil. They decided to set up test boxes, empty half-pint milk cartons, with increments of fertilizer increase of one-sixteenth ounce. They weighed a cup of soil and found it weighed about six ounces. They decided to fill one fourth of each cup with sand for drainage. They divided the depth of the cup (2") by four to determine this depth. This left them with one and one-half inches of soil. They proceeded to determine the weight of this soil so they would know how much fertilizer to add to each cup. Some of the pupils calculated this weight by finding one half of six ounces, then one half of the three ounces, and adding the two answers to get four and one half ounces. They drew diagrams to prove their answers. Some children used the following ratio to find the answer: $2"{:}6\text{ oz.}$ $1\frac{1}{2}"{:}x\text{ oz.}$

*Adapted from a practice submitted by Lynn Olson, Lafayette School, Hammond.



Mathematics helps these fifth grade "scientists" figure out how long it would take to go to all the different planets in our solar system at the rate of 60 miles per hour. (Hammond Public Schools)

The pupils now designed a chart to keep a record of the amount of fertilizer they would add to the various cups of soil. For each cup they calculated the ratio of fertilizer to soil. In this process they learned how to divide first by a fraction with a numerator of one and then by a fraction with a numerator of more than one.

Integration with the Social Studies in Grade 5*

To begin a fifth grade study of the United States, a large United States map was projected and traced on the bulletin board. After it was fully drawn, it was measured and the distances were compared with corresponding distances on a similar map in the textbook. On the basis of the scale of the map in the book, the class computed a scale for the enlarged map.

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*A practice submitted by Naomi Wall, Washington School, Vincennes.

Their knowledge of mathematics helps these intermediate grade children draw a map of Indiana to scale.
(Hammond Public Schools)

Since this unit began in the fall semester, each child was eager to tell about his summer vacation trip. To create interest in and to further initiate this unit, each child was given a small slip of colored paper on which he wrote the name of the place he had visited during the summer. Then, using the map in the text book, he found how far this place was from his home. He first measured the distance in inches on the map and

then converted this to miles. He then converted this back into inches, this time using the scale of the enlarged bulletin board map. He then measured the distance on the large map and pinned his name and summer vacation spot on the appropriate place on the map.

Mathematics helps these intermediate grade children in their social studies work. (Hammond Public Schools)

The class made several time lines to show the relationship between events in a history unit. One time line showed the events in the period of exploration of the New World, from 1200 to 1750. The following activities connected with developing the time line were largely arithmetical:

1. Finding the number of years included in the period to be shown

2. Measuring the line used as a base for recording the dates

3. Determining the length of each interval and then finding the number of intervals needed to include the whole period

4. Finding how many inches on the line would be used for each interval

5. Adding to find each successive date for the intervals

6. Estimating where an event should go if it fell between the dates shown on the intervals.

Generalizations

1. Unified learning experiences enable children to perceive quantitative relationships within a meaningful context.

2. Quantitative information can be identified in all areas of the curriculum.

3. Different curricular activities are rich in the application of a wide variety of quantitative procedures.

4. Curricular areas other than mathematics can be used to enrich and extend numerical concepts and mathematical terminology.

5. Curricular areas such as science and social studies help to develop appreciation of the significance and importance of mathematics in our daily lives.

6. Curricular areas other than mathematics can stimulate the need to learn and practice a variety of computational procedures.

USING UNITS OF WORK IN MATHEMATICS

A mathematics program may be enriched by a cooperatively planned unit of experiences related to a topic of both mathematical and social significance. Such units of work provide learning experiences which will help children achieve accepted goals. Teachers find that such units of work enable them to cut across subject-matter lines and to relate other curriculum areas to mathematics. Functional use of subject matter and skills is an important outcome of such an approach.

Units of work enable teachers to meet individual differences more satisfactorily. Since pupils have a variety of purposes, activities, interests, and abilities, it can be assumed that each child can make a contribution of some kind to the total group effort. Hundreds of activities, such as those related to problem solving, construction, field trips, and creativity, are possible sources of activities that can be carried on in a unit of work in mathematics. The ongoing daily activities of children are the best sources for units of work. Such topics as the calendar, money, time, linear measurement, graphs, and maps are but a few examples of the many possible materials that may be used in broad units of work in mathematics. If the teacher is really interested in promoting valuable kinds of group and individual learning where effective techniques of thinking, the perception of relationships, and the methods of locating, organizing, and evaluating information are given specific attention, the unit of work will become a part of the mathematics program.

Unit in Mathematics in Grade 6*

Designed originally to stimulate quantitative thinking and to develop frames of reference, this unit exploded with scintillating appeal upon the imagination of the children. This unit, which was planned to give the children a better understanding and a greater appreciation of large numbers, began very simply.

"How much is a billion dollars?" they were asked. "If we pretended a $2\frac{1}{2}$ x 6 inch piece of paper was a thousand dollar bill, how many would we need to make the billion? How big would the stack of bills be?" Their eyes just grew bigger and bigger and bigger.

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*Adapted from a practice submitted by Joseph Cailles, S. Ellen Jones School, New Albany.

The children were shown how to fold and rip a newspaper into halves, fourths, eighths, and sixteenths. The latter were measured to six-inch strips and cut. The resultant bills were then to be stacked into a pack of one hundred.

What does a billion look like? These children found out as part of their unit of work dealing with large numbers. (New Albany-Floyd County Consolidated Schools)

In five days' time, an agreed upon deadline, $100,000,000 (one hundred million) had been minted, and the original class of 43 boys and girls had been stretched to include over a hundred children from grades 3 to 6. "How much have we got?" became the question of the day.

It was during the planning of the presentation stage that the idea of an arithmetic jamboree evolved. Signs, tables, and graphs had to be designed. Problems had to be solved. Complex and strange as these were, the sixth grade children attacked them with rare gusto.

How many pieces of paper did we actually have?

How much would each bill have to represent if we wanted to depict a billion with what we had?

How much would we have if each was a $10 bill? A $100 bill?

How high would a stack of $1,000 bills be which totaled $1,000,000?

How high would the stack be if the full billion were measured?

How many feet is this? How many yards?

A hundred million is what part of a billion?

Are there any other ways of showing a concept of a very large number? A million?

How large an area would we need to show 1,000,000 square inches?

A thousand inches is how many feet? How many yards?

Can we compare decimals and fractions to see which would be easier to use in solving certain problems?

How many pages of newspaper 22 x 30 inches would we need to cover an area of one million square inches?

How many pages would each child in the class have to bring?

Would we have room in our gymnasium to show this area?

What about the idea that our sun's volume is 1,000,000 times greater than our earth's?

What is volume?

What is the volume of a toy block?

Could we create an imaginary tank to hold a million of these blocks? How?

The above questions are just a sample of the many questions raised by the children, and herein lay a most startling discovery! The children had an opportunity to apply many of their arithmetic skills in these problem situations, and this illustrated to them beyond all words, "Why arithmetic?" And they loved it!

Outline Unit in Mathematics in Grade 4*

Unit: The Measurement of Time

I. Specific Objectives

To help children:

A. To develop an awareness of the importance of the ways of measurement of time in their daily lives.

B. To understand how people of long ago measured time.

C. To appreciate that many peoples of the world have contributed to the accurate measurement of time.

II. Suggested Activities

A. Approach

1. Make a list of activities which children do by the clock.
2. Discuss ways of telling time in the children's homes.
3. Discuss how the children might tell time if all clocks in their homes stopped.

B. Developmental

1. List children's understandings and questions concerning the measurement of time.
2. Observe that the sun's place in the sky divides the day into three parts.
3. Study the shadow of the flagpole at various times of the day to discover how shadows helped early man in measuring time.
4. Discover the location of sundials in the community.
5. List reasons why a sundial is not a satisfactory clock.
6. Read to learn ways of telling time in various parts of the world long ago: fire clocks, water clocks, sand clocks, etc.
7. Make a simple water clock with paper cups and straws.
8. Experiment with candles marked with stripes to discover how a burning candle could measure time.
9. Learn the various sources of power that run clocks today.
10. Give reports on famous clocks of the world today: Big Ben, Naval Observatory clocks, Clock of Berne, Switzerland.

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*Adapted from a practice submitted by Doris Rushton, General Consultant, Indianapolis Public Schools.

11. Display the parts of a broken clock in order to understand how it works and to appreciate the skill of a clock and watch repairman.
12. Compare the accuracy of clocks today with those of long ago.
13. Report the type of time piece, and how measurement of time is used in sports, cooking, industry, etc.
14. Write an imaginary story of the problems encountered in "a timeless" 24 hours.
15. Discover how atomic energy will measure time in the future.

C. Culminating activities

1. Arrange an exhibit of ways of measuring time now and long ago.
2. Invite parents or other children to see the exhibit and hear the results of the research.

III. Evaluation

A. Have children developed an awareness of the importance of the measurement of time in their daily lives?

B. Do children understand and can they explain some of the ways of telling time long ago?

C. Have the children gained some feeling for the slow development of accurate time devices and the contributions of many individuals and cultures?

Generalizations

1. Units of work provide continuity in the quantitative learning experiences of children.

2. Units of work in mathematics provide a range of activities to meet the individual needs and interests of children.

3. Excellent opportunities to promote problem-solving behavior are found in units of work in mathematics.

4. Enriched units of work in mathematics often grow out of the current activities of children.

5. The development of skills in securing, organizing, and evaluating information are promoted in units of work in mathematics.

6. Units of work give vitality to the mathematics program.

7. Units of work in mathematics provide a functional way for children to use subject matter and skills.

SUMMARY

Teachers are becoming increasingly concerned with the development of a broader concept of mathematics in the elementary school. This concern is being reflected in their reported classroom practices. Mere rote, mechanical computation is giving way to programs which stress quantitative understanding and meaning.

Almost all of the reporting teachers indicated widespread use of a variety of instructional materials, materials centers, and bulletin boards in their mathematics programs. It seems evident that the single textbook per child method of instruction is being replaced by the learning laboratory method.

Over half of the reporting teachers indicated promising practices in promoting growth in quantitative vocabulary and related reading skills in mathematics. Such expressions of interest and concern in this phase of a modern program are bound to bring about satisfying results.

The practices reported for problem solving were somewhat disappointing in that the majority of them relied heavily upon either the "recipe" or "cue word" approach. This is obviously an aspect of the program in which teachers need to do more careful thinking and evaluating of present practices if quantitative reasoning is their goal.

The fact that only six schools reported practices dealing with the history and development of our number system suggests that this is still not considered a needed aspect of most mathematics programs in the elementary school. It seems strange that mathematics is the only science that does not make use of its historical heritage in developing a better understanding of its present and future status. A reexamination is needed of the role that history can play in a mathematics program.

Perhaps the most challenging and promising practices were reported as units of work and/or of integration of mathematics with other subject fields. Every school involved in this study reported one or more such practices. Significantly, many of the teachers pointed out that mathematics does permeate the entire curriculum. Certainly, the subject serves and is served best when the alert teacher takes this into account.

Evidences of creativity, imagination, and enthusiasm were to be found in many of the reported practices. This is encouraging for, slow though the change may be, a modern program in elementary school mathematics is taking shape and is finding direction in the hands of concerned, interested, and capable teachers.

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